TORSIONAL STIFFNESS
CALCULATION AND VALIDATION

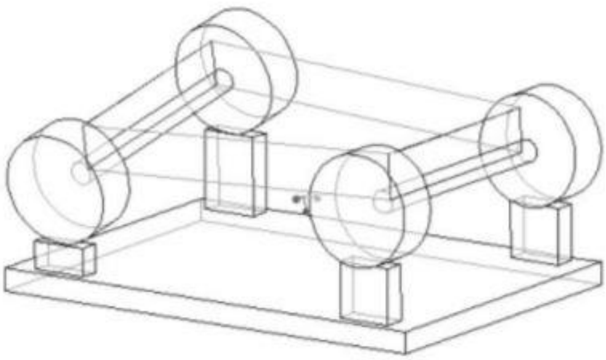
INDEX

Literature Review

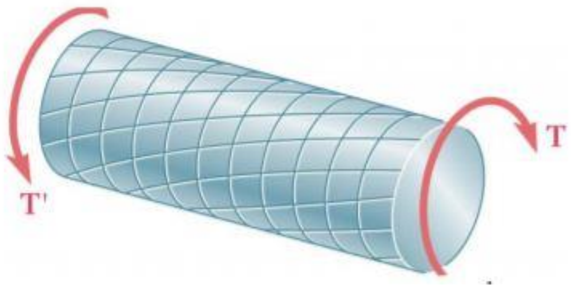
What is torsional stiffness or torsional rigidity? Stiffness is the resistance to bending or flexing while Torsional stiffness/rigidity is the resistance to twisting. It is the amount of torque required to twist an object through one degree. It has units of N-m/deg or ft-lb/deg. While calculating wheel loading under various driving conditions, it is important to recognize that all the calculations of suspension parameters are based on the assumption that the chassis of the car is rigid and has infinite stiffness, that is, it does not bend or twist under any driving condition. But in actual the chassis isn't infinitely stiff and does bend and twist under loading.

A performance vehicle must have adequate chassis torsional stiffness around the x-axis, that is, longitudinal axis along the length of the vehicle. Thinking of the chassis as a large spring connecting the front and rear suspensions, if the chassis torsional spring is weak, attempts to control the lateral load transfer distribution will be confusing at best and impossible at worst. This is because a flexible spring adds another spring to an already complex system. Predictable handling can best be achieved if the chassis is stiff enough to be safely ignored. Hence it is important to make sure the chassis is stiff enough. Now the question that follows is 'how stiff is stiff enough?' We'll come back to this question in later part of this document.

Torsional deflections of a car chassis result from forces induced in the frame by the suspension. If the chassis is not stiff enough, the suspension loads create a tendency for the frame to twist about the longitudinal axis running down its length. Excessive deflections make control of lateral load transfer, one of the primary functions of a car's suspension, difficult. Therefore, the torsional rigidity of the frame is critical to the dynamic performance of the car. The figure below shows the twisting of a chassis.



The first step in any engineering problem is to break it down into its simplest form and fit it to a theoretical model. Torsion, in its simplest form, is defined on a solid circular member. From Mechanics of Materials (or Solid Mechanics), the applied torque couple, T, is proportional to the angular deflection θ. The constant of proportionality is known as the torsional rigidity and is related to geometric and material properties of the member in torsion.



$$T = k * \theta$$

Where T is the applied torque, θ is the angular deflection, and k is torsional rigidity.

$$k = J*G/L$$

Where J is the second moment of area, G is the modulus of rigidity (or shear modulus), and L is the length of the member. For a simple beam, such as the one in Figure, 'k' can be arithmetically determined because 'J', 'G', and 'L' are all easily known. However, for a more complicated beam shape, such as an FSAE spaceframe, the second moment of area is more challenging to calculate.

Instead, an experiment can be run in which different torques are used to measure angular deflections. Theory says that these values plotted against each other should be linear as long as the experiment remained within the elastic range of the material. A linear trend analysis is used to determine torsional rigidity, which is no longer constant because 'J' becomes a function of position for irregular beams.

The biggest effect of torsional deflection has been found to be the effect it has on lateral load transfer distribution between the front and rear axle. During a steady state turn an infinitely rigid chassis will cause the front and rear roll angle to be identical, as is assumed when suspension design calculations are performed. Allowing for twist in the chassis will redistribute some amount of weight transfer between the front and rear tires, causing these values to deviate from their designed values. Because of these effects, the vehicle will handle more predictably if the chassis is stiff enough, relative to the suspension roll stiffness, that the twist can be ignored. To achieve this goal, most racecars design to be greater than the suspension roll stiffness. Finding a standard value against which a chassis can be compared would be helpful in the design of vehicles, but as of current, no standard criterion exists.

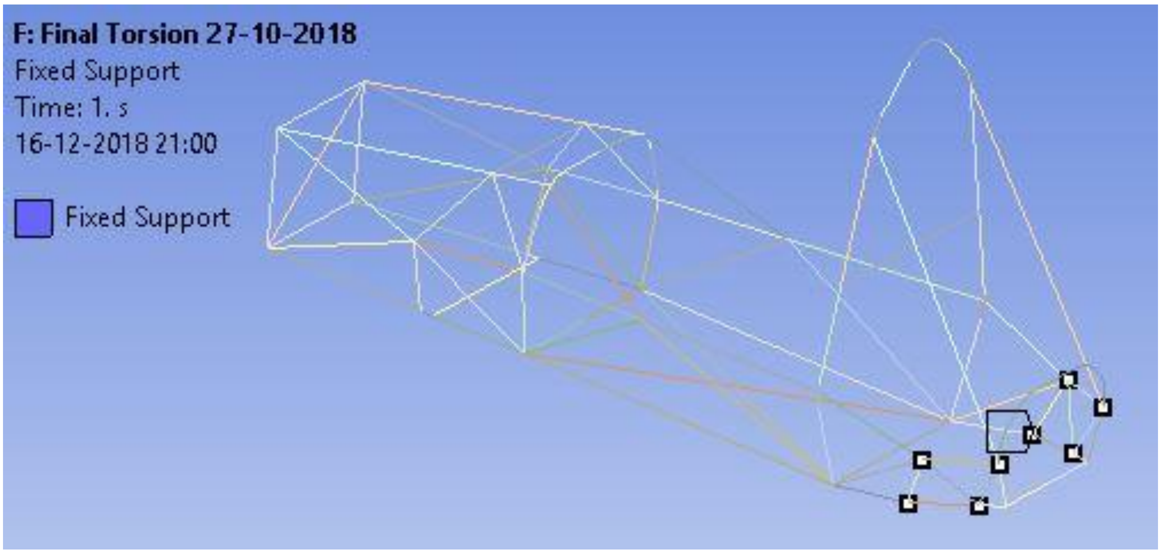
As well as the effects above, twist in the chassis has also been theorized to have negative effects on the fatigue of the vehicle, suspension characteristic changes caused by relative hard point displacement, and transient response time of the vehicle as a whole to driver inputs. All of these effects can be easily mitigated by increasing the stiffness of the vehicle, but this has the direct response of increasing vehicle weight. For this reason, it is desired to find the lowest value of torsional stiffness that will not decrease the performance of the car. The target value that was set for the torsional rigidity was 1400 Nm/degree. This value was decided as the target value based on various documents online and seminars attended.

Simulation Method

Boundary Conditions-

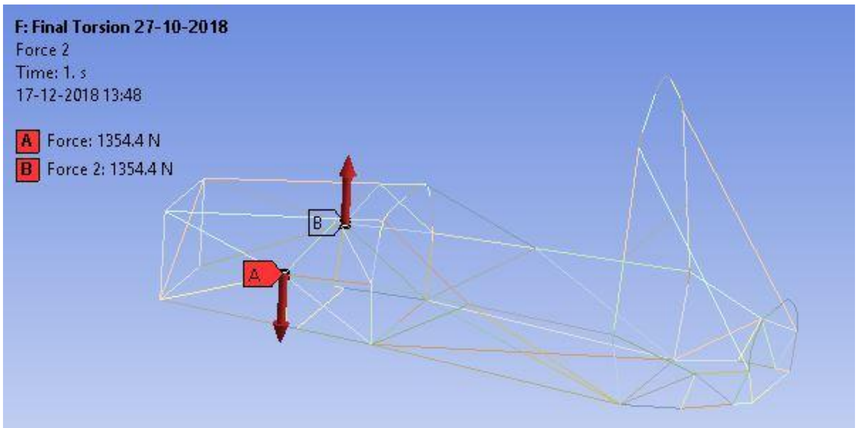
1. Fixed Supports-

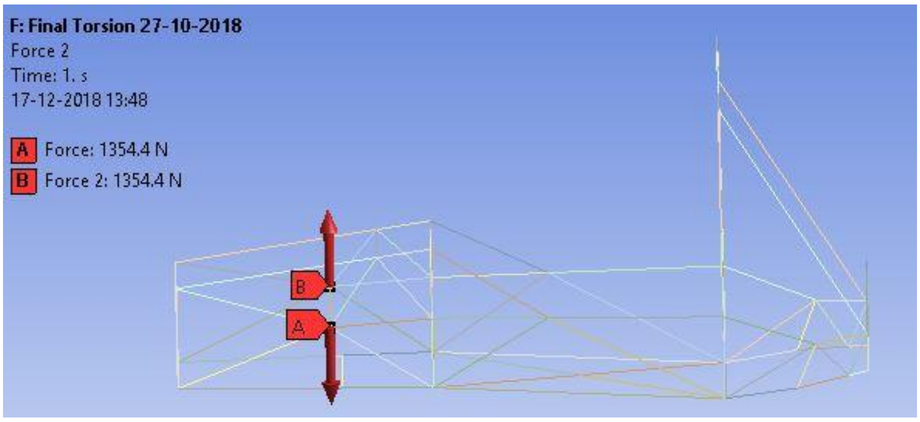
   Rear suspension points as shown,



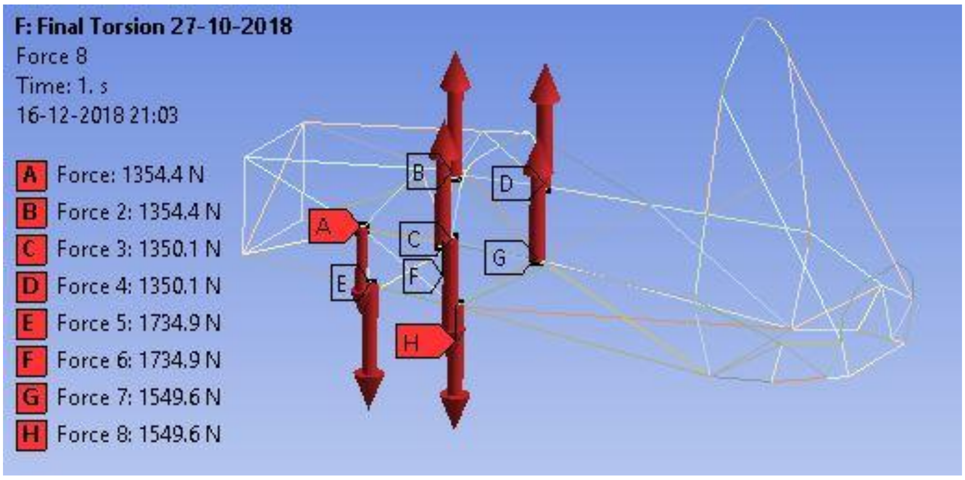
2. Forces-
   - First, torque due to bump force on tires is taken as 3000Nm.
   - This torque is applied through forces acting on 4 front suspension points, coming through tires and then through A arms.

- These 4 front suspension points are say upper front (UF) 1, upper rear (UR) 2, lower front (LF) 3, lower rear (LR) 4.
- Hence, at each front suspension point equivalent amount of force acts which lead to total of 3000Nm torque.
- At each suspension point force acts in upward direction and equivalent force acts in opposite direction at corresponding point on other side as shown,





- Similarly, at other 3 points also torque acts as shown, to get total amount of 3000Nm torque.



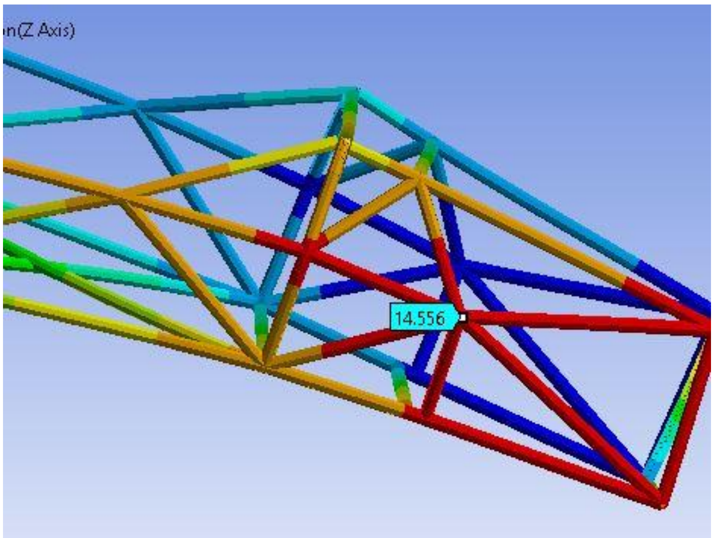
- Distances between corresponding points are

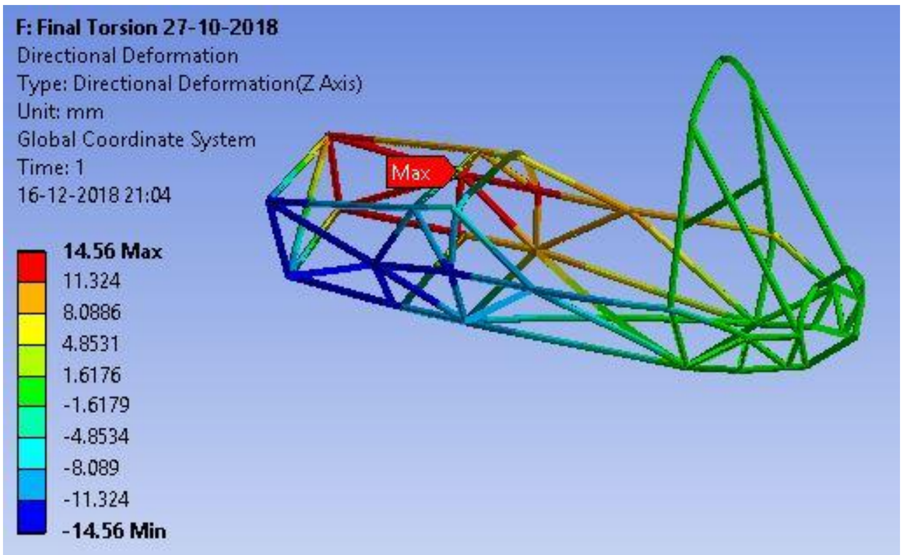
UF = 553.768mm=d1
UR = 555.53mm =d2
LF = 432.29mm =d3
LR = 483.998mm=d4

- These forces are calculated as follow,
  Total torque = torque at UF + torque at UR + torque at LF + torque at LR
  3000 = F1*d1 + F2*d2 + F3*d3 + F4*d4
  Assume,
  F1*d1 = F2*d2 = F3*d3 = F4*d4 = 3000/4 = 750 Nm
  Hence, F1*d1 = 750
  F1 = 750/d1
  = 750/0.553768m
  F1 = 1354.3577N

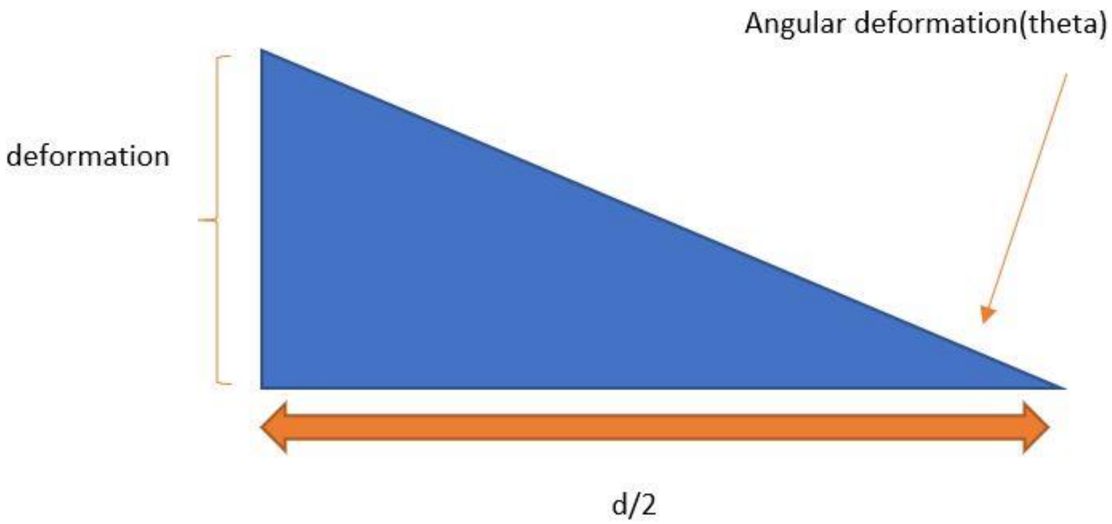
  Similarly,
  F2 = 750/d2=1350.0621N
  F3 = 750/d3=1734.9464N
  F4 = 750/d4=1549.59318N

3. After performing FEA, we got directional deformations in Z direction at each suspension point as shown,





- UF=14.56mm
  UR=11.55mm
  LF=11.5mm
  LR=10.082mm



- Hence angular deformation is found at each point as
- $\tan^{-1} \quad deformation/distance$
- And, torsional rigidity= (Total torque)/angular deformation
  1. UF-
     Theta=$\tan^{-1} \quad 14.56 * 2/553.768$= 3.0101degree
     T.R. = 3000/3.0101 =996.6446 N.m/degree

  2. UR-
     Theta=$\tan^{-1} \quad 11.55 * 2/555.53$= 2.38109degree
     T.R. = 3000/2.38109=1259.9271 N.m/degree

  3. LF-
     Theta=$\tan^{-1} \quad 11.5 * 2/432.29$= 3.0455degree
     T.R. = 3000/3.0455= 985.0455N.m/degree

  4. LR-
     Theta=$\tan^{-1} \quad 10.082 * 2/483.998$= 2.3856degree
     T.R. = 3000/2.3856= 1257.5248N.m/degree

- Average T.R.= 1124.7849N.m/degree
- Average Theta =2.70557degree

4. Similarly,

We calculated T.R. and Theta average for different torques from 1000Nm to 10000Nm as shown in following calculations,

- Distances-
  UF = 553.768 mm = 0.553768 m = d1
  UR = 555.53 mm = 0.55553 m = d2
  LF = 432.29 mm = 0.43229 m = d3
  LR = 483.998 mm = 0.483998 m = d4

  Torque (total) = F1*d1 + F2 * d2 + F3*d3 + F4*d4
  - Assumption – F1*d1 = F2*d2 = F3*d3 = F4*d4

1) Torque (total) = 3000Nm

- Forces-
  F1*d1 = 3000/4 = 750 Nm
  Hence,
  F1 = 750/d1 = 750/0.553768 = 1354.3577 N
  F2 = 750/d2 =1350.0621 N
  F3 = 750/d3 =1734.9464 N
  F4 = 750/d4 =1549.59318 N

- Deformations-
  UF = 14.56 mm
  UR = 11.55 mm
  LF = 11.5 mm
  LR = 10.082 mm

- Torsional Rigidity (T.R.) –

  i.UF –

  $\theta = \tan^{-1} \frac{14.56*2}{553.768}$ = 3.0101

  T.R. = $\frac{3000}{\theta}$ = 996.6446 Nm/degree

  ii. UR –

  $\theta = \tan^{-1} \frac{11.55*2}{555.53}$ = 238109

  T.R. = $\frac{3000}{\theta}$ = 1259.9271 Nm/degree

  iii. LF –

  $\theta = \tan^{-1} \frac{11.5*2}{432.29}$ = 3.0455

  T.R. = $\frac{3000}{\theta}$ = 985.0431 Nm/degree

  iv.LF –

  $\theta = \tan^{-1} \frac{10.082*2}{483.998}$ = 2.3856

  T.R. = $\frac{3000}{\theta}$ = 1257.5248 Nm/degree

- Average (Final) T.R. = $\frac{996.6446+1259.9271+985.0431+1257.5248}{4}$
  T.R = 1124.7849 Nm/degree
- Average θ = 2.70557 degree

We did this first for 3000Nm torque as generally due to bump force, this much of torque acts. We did similar calculations for all torques from 1000 to 10000 Nm torque as below,

2) Torque (total) = 1000 Nm

Hence, F1d1 = F2d2 = F3d3 = F4d4 = 1000/4 = 250 Nm

SR .No.	Dist.	Forces (N)	Deform. (mm)	θ (degree)	T.R. (Nm/degree)
1	d1	451.4525	4.8532	1.00417	995.8473
2	d2	450.0207	3.857	0.79554	1256.9938
3	d3	578.3154	3.8277	1.0145	985.7072
4	d4	516.53106	3.357	0.7947	1258.3364

Average (Final) T.R. = 1124.221175 Nm/degree

Average θ = 0.9022275 degree

3) Torque (total) = 2000 Nm

Hence, F1d1 = F2d2 = F3d3 = F4d4 = 2000/4 = 500 Nm

SR .No.	Dist.	Forces (N)	Deform. (mm)	θ (degree)	T.R. (Nm/degree)
1	d1	902.90518	9.7065	2.00774	1257.89
2	d2	900.0414	7.7133	1.5906	1257.3871
3	d3	1156.6309	7.6647	2.0309	984.785
4	d4	1033.0621	6.7172	1.5899	1257.9407

Average (Final) T.R. = 1189.5007 Nm/degree

Average θ = 1.804785 degree

4) Torque (total) = 4000 Nm

Hence, F1d1 = F2d2 = F3d3 = F4d4 = 4000/4 = 1000 Nm

SR .No.	Dist.	Forces (N)	Deform. (mm)	θ (degree)	T.R. (Nm/degree)
1	d1	1805.8103	19.413	4.0105	997.3818
2	d2	1800.0828	15.421	3.17769	1258.776
3	d3	2313.2619	15.332	4.0574	985.853
4	d4	2066.1242	13.44	3.17879	1258.3387

Average (Final) T.R. = 1125.0873 Nm/degree

Average θ = 3.606095 degree

5) Torque (total) = 5000 Nm

Hence, F1d1 = F2d2 = F3d3 = F4d4 = 5000/4 = 1250 Nm

SR .No.	Dist.	Forces (N)	Deform. (mm)	θ (degree)	T.R. (Nm/degree)
1	d1	2257.2629	24.266	5.00881	998.2867
2	d2	2250.1035	19.275	3.969573	1259.58132
3	d3	2891.5774	19.165	5.067012	986.774848
4	d4	2582.6553	16.8	3.9712	1259.06527

Average (Final) T.R. = 1125.927045 Nm/degree

Average θ = 4.4892915 degree

6) Torque (total) = 6000 Nm
   Hence, F1d1 = F2d2 = F3d3 = F4d4 = 6000/4 = 1500 Nm

SR .No.	Dist.	Forces (N)	Deform. (mm)	θ (degree)	T.R. (Nm/degree)
1	d1	2708.7155	29.119	6.0035	999.4170
2	d2	2700.1242	23.114	4.7568	1261.35217
3	d3	3469.8928	22.996	6.0729	987.9958
4	d4	3099.1863	20.16	4.762093	1259.9499

Average (Final) T.R. = 1127.1787 Nm/degree

Average θ = 5.39882325 degree

7) Torque (total) = 7000 Nm
   Hence, F1d1 = F2d2 = F3d3 = F4d4 = 7000/4 = 1750 Nm

SR .No.	Dist.	Forces (N)	Deform. (mm)	θ (degree)	T.R. (Nm/degree)
1	d1	3160.16815	33.973	6.995	1000.714796
2	d2	3150.1449	26.987	5.5493	1261.42
3	d3	4048.2083	26.819	7.073038	989.6736
4	d4	3615.71742	23.523	5.55187	1260.8357

Average (Final) T.R. = 1128.161024 Nm/degree

Average θ = 6.2923 degree

8) Torque (total) = 8000 Nm
   Hence, F1d1 = F2d2 = F3d3 = F4d4 = 8000/4 = 2000 Nm

SR .No.	Dist.	Forces (N)	Deform. (mm)	θ (degree)	T.R. (Nm/degree)
1	d1	3611.6207	37.143	7.6404	1047.0639
2	d2	3600.1656	29.543	6.0711	1317.7103
3	d3	4626.5238	29.111	7.67059	1042.9439
4	d4	4132.2484	25.651	6.0505	1322.1941

Average (Final) T.R. = 1182.478058 Nm/degree

Average θ = 6.8581475 degre

9) Torque (total) = 9000 Nm
Hence, F1d1 = F2d2 = F3d3 = F4d4 = 9000/4 = 2250 Nm

SR .No.	Dist.	Forces (N)	Deform. (mm)	θ (degree)	T.R. (Nm/degree)
1	d1	4063.0733	43.656	8.96	1004.464
2	d2	4050.1863	34.685	7.11778	1264.439
3	d3	5204.8393	34.504	9.0698	992.3035
4	d4	4648.7795	30.17	7.106395	1266.4649

Average (Final) T.R. = 1131.9178 Nm/degree

Average θ = 8.06349375 degree

10) Torque (total) = 10000 Nm
Hence, F1d1 = F2d2 = F3d3 = F4d4 = 10000/4 = 2500 Nm

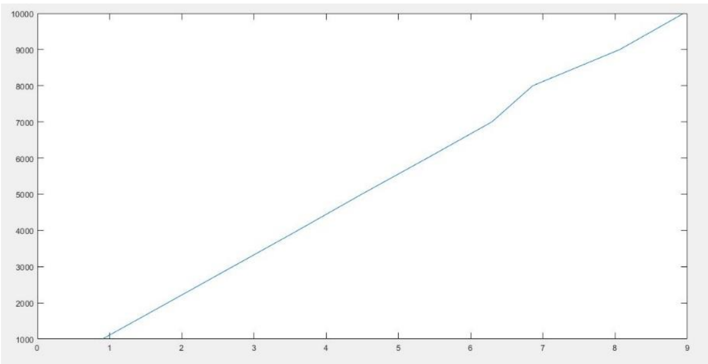
SR .No.	Dist.	Forces (N)	Deform. (mm)	θ (degree)	T.R. (Nm/degree)
1	d1	4514.5259	48.533	9.9419	1005.843953
2	d2	4500.20701	38.5	7.8912	1267.22128
3	d3	5783.15482	38.295	10.046989	995.3229
4	d4	5165.3106	33.629	7.91135	1264.006752

Average (Final) T.R. = 1133.09872 Nm/degree

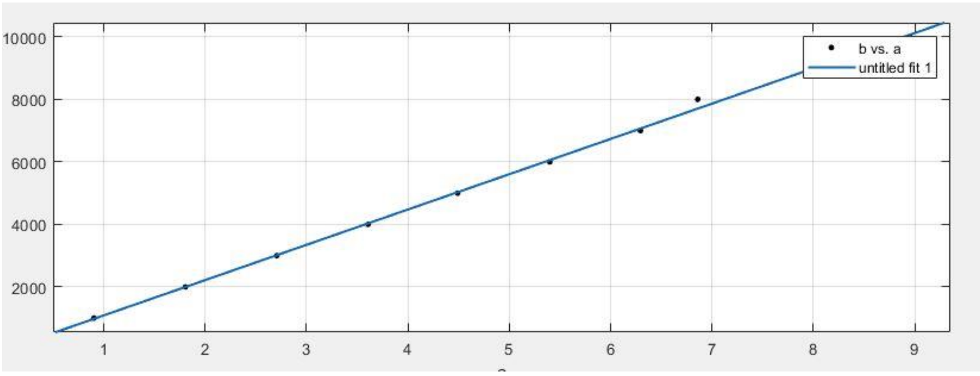
Average θ = 8.94785975 degree

- We observe that all T.R. values are nearly constant.
- Average of all T.R. values is 1139.235542Nm/degree
- Average of all angular deformations is 4.235755719 degree

5. Then we plotted graph between different torques and corresponding angular deformations as shown,



- After plotting best curve fit using matlab, we found its slope which is our final T.R. value



Results

Linear model Poly1:
    f(x) = p1*x + p2
Coefficients (with 95% confidence bounds):
    p1 =   1129 (1096, 1162)
    p2 =   -40.11 (-222.4, 142.2)

Goodness of fit:
  SSE: 1.058e+05
  R-square: 0.9987
  Adjusted R-square: 0.9986
  RMSE: 115

Hence, slope of above curve fit is 1129.
Therefore, FINAL TORSIONAL RIGIDITY OF CHASSIS IS 1129Nm/degree BY FEA.

- Then we found T.R. practically by RIG and cross checked the results

Experimental Method

Apparatus

The goal of the test stand was to constrain and torque the frame to measure the torsional deflection. The test stand included 5 major components: the front and rear fixtures, the front pivot point, the beam, and the weights used for providing torque.

The rendered CAD model and experimental model of the system are shown below.

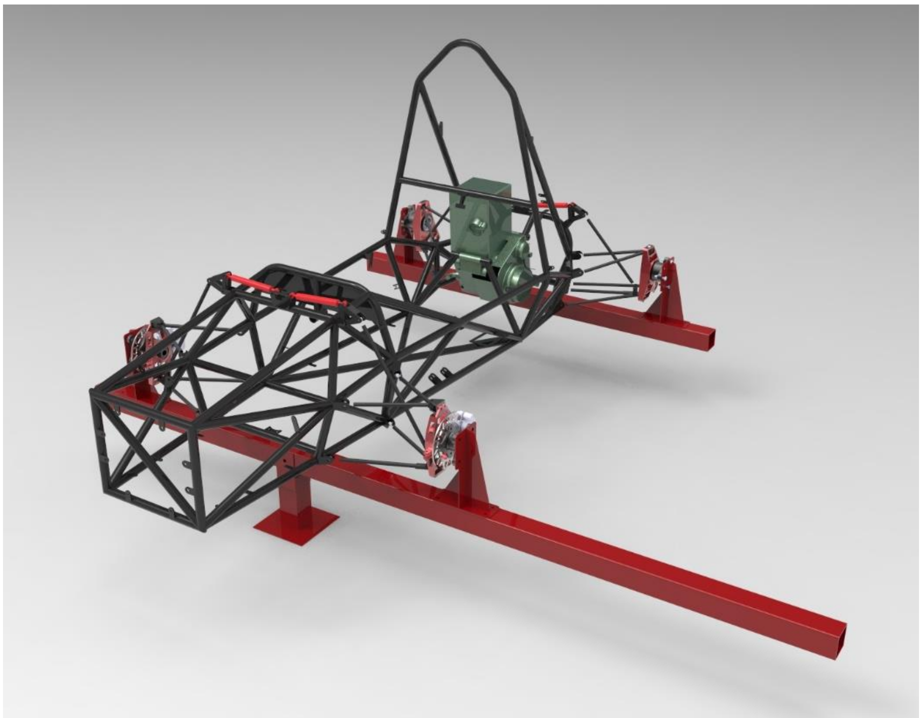


Image 1

The rear fixture was bolted to the rear wheel hub through the four M12 lug bolts. The fixture was bolted to a steel square section beam with eight M8 bolts. The two rear fixtures completely constrained the rear part of the vehicle in all degrees of translation and two degrees of rotation. The beam was fixed to the ground. The rear fixture assembly is shown in images below.

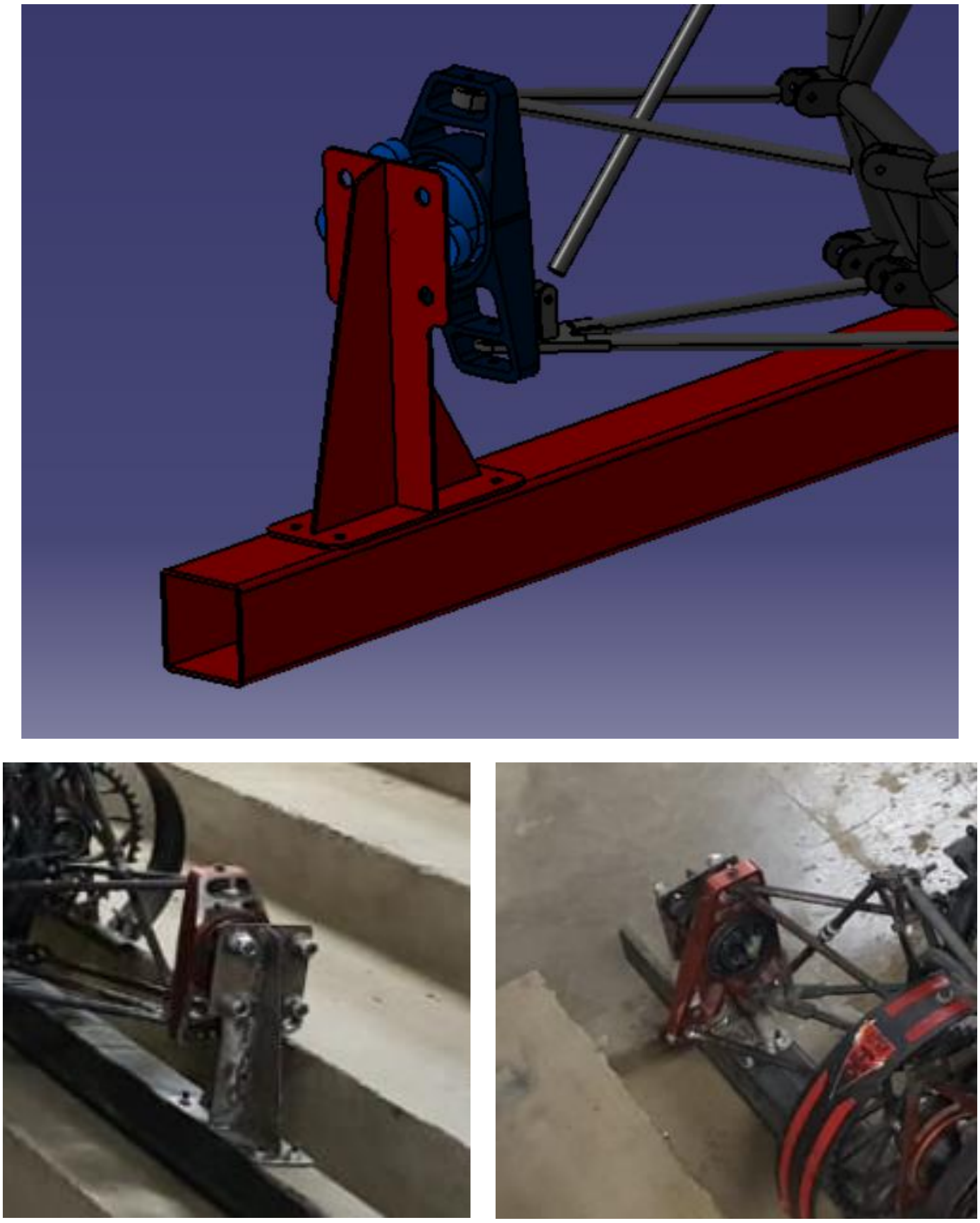
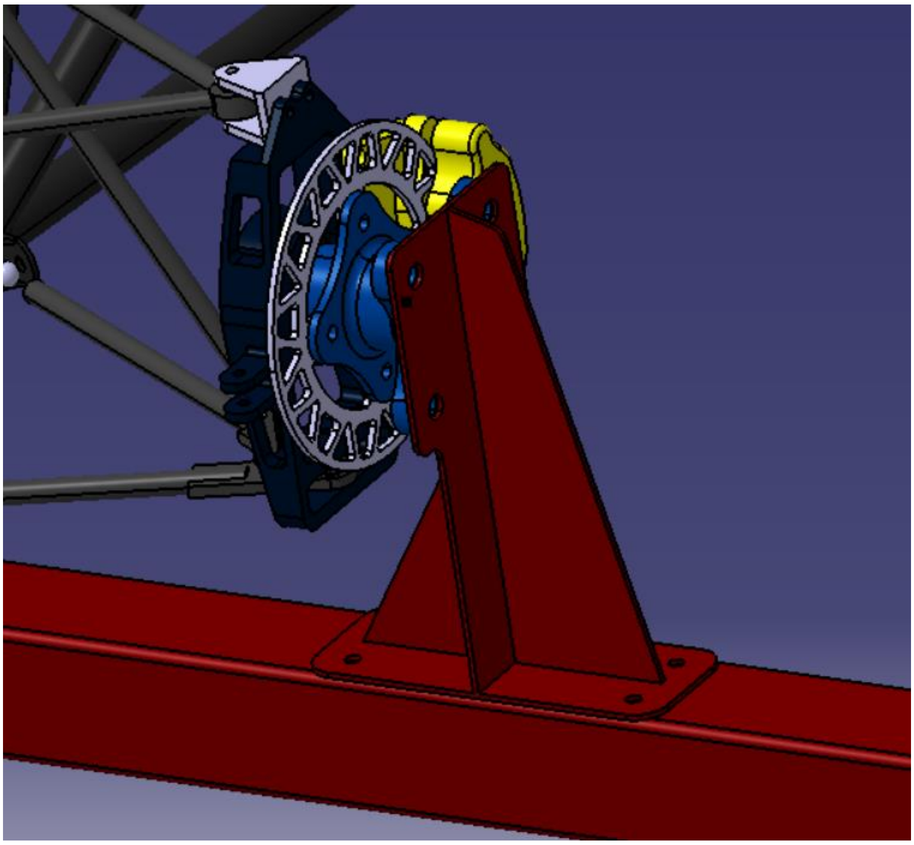


Image 2

Using two rear end fixtures, yawing and rolling motion is constrained. The only motion that has not been constrained is pitching of the frame, which the front pivot point constrains. The dimensions of steel square section pipe used was 72mm X 3mm. The fixture plates are 3mm Mild Steel.

The front fixtures were bolted to the front hub in a similar manner as the rear fixtures and the same was bolted to a lower beam or lever arm used for applying torque on the frame. All fixture plates used were laser cut and accuracy was maintained. Image of front fixture assembly is shown below.



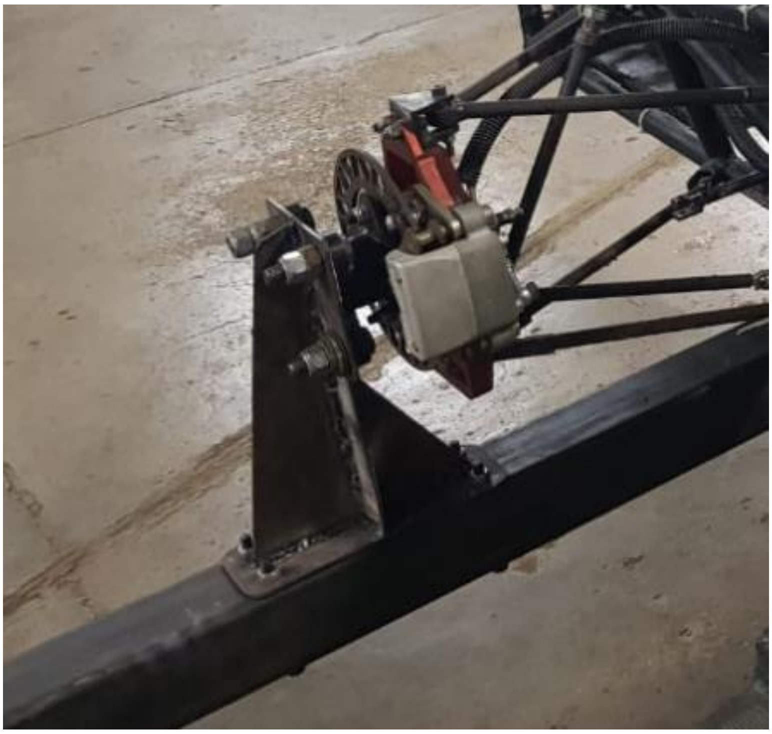


Image 3

It is critical to the experiment that the left and right front fixtures are connected to each other via a rigid beam to apply a force couple along the front axle. The front pivot point was used to constrain rotation at the instantaneous front roll center. A photograph of the front pivot point is shown below.

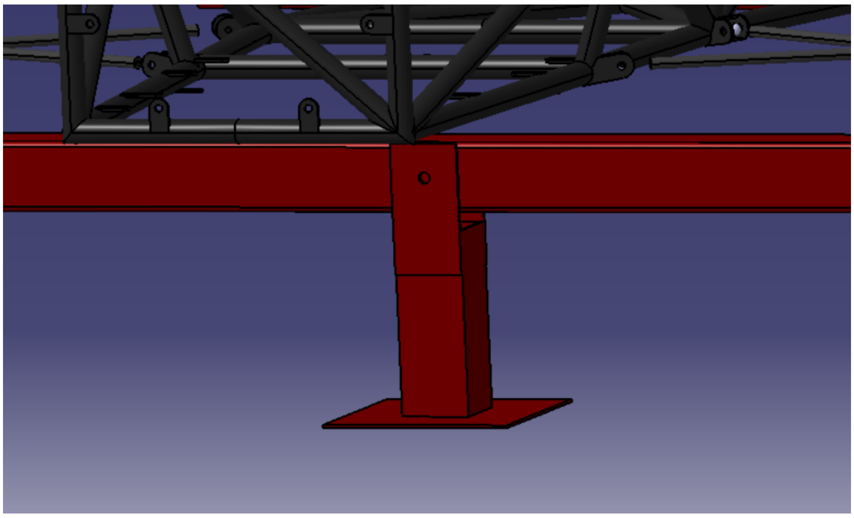
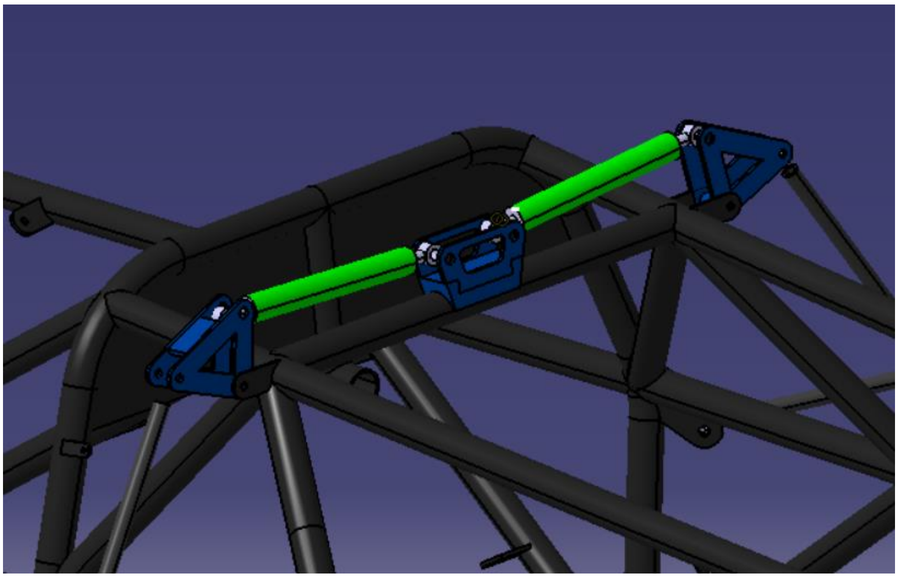


Image 4

The front pivot was made of same cross section square pipe. An Allen bolt was used as the pivot to ensure a smooth rotation and adequate clearance was kept between the beam and the pivot.

The front and rear suspension spring were replaced with solid links with same dimensions in terms of length. The image for same is shown below. This was done to take out the effect of suspension roll stiffness out of the picture. Another reason was the physical test could possibly damage the dampers.



The torque applied were with the help of weights at different lengths from the pivot.

Instrumentation

The torque was measured by applying the load with the help of weights at specific distances from the pivot point.

To measure the deflection of the frame, deflection gauges weren't used but instead low cost laser pointers and projection white paper were used. The laser pointers give the necessary resolution to measure torsional deflections of less than a degree given a screen that is far enough away. The screen was kept at a distance of 3m from the pivot. Standard office laser pointers were mounted to the frame at several points of interest. The mounting locations of the lasers are as shown in the image below. Zip ties were used to mount the lasers to the frame. The zip ties can be tightened down on the button of the laser pointer, so the laser pointer was always on for the duration of the experiment.

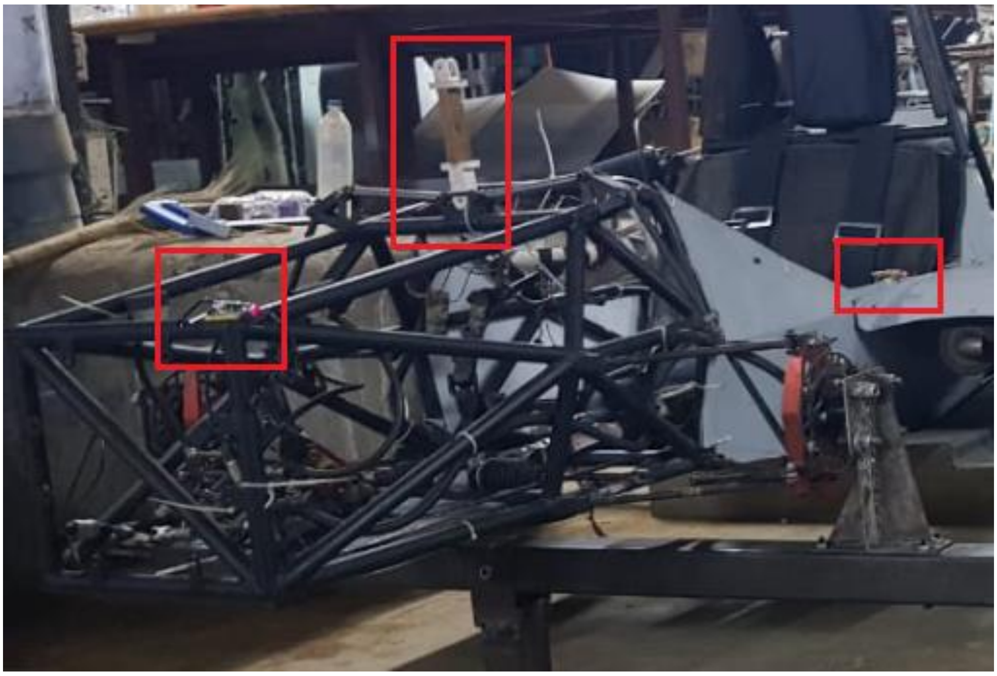


Image 5

There were lasers placed at different positions of interest along the length of the chassis at the locations shown in the figure below. The first test had 3 Lasers as shown in red squares to calculate the deflection in the parts forward of the plane where the rear part frame is fixed. An additional was used at the rear bulkhead because there was slight twist in the rear frame as well, this is explained in later part of the document.

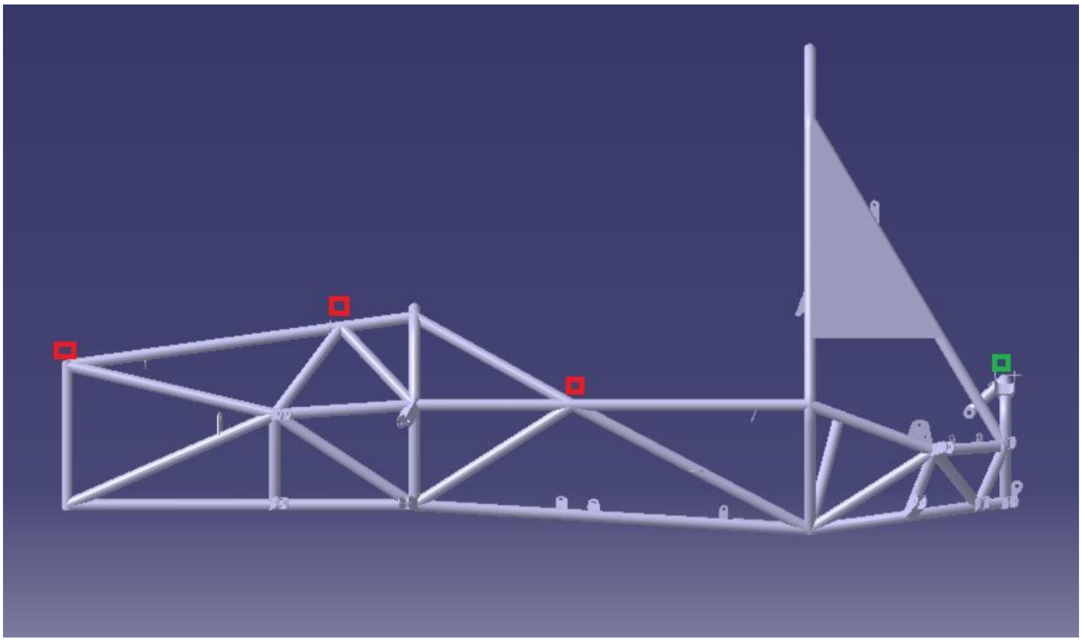


Image 6

The laser at the spring mount region as shown in the middle region was in the same lateral plane of front axle. The laser projected location is as shown in the image below on the screen which is at a distance of 3 meters from the central longitudinal plane of the vehicle. During the experiment the position of laser was marked on the screen which was a white paper with the help of a pinpoint marker.

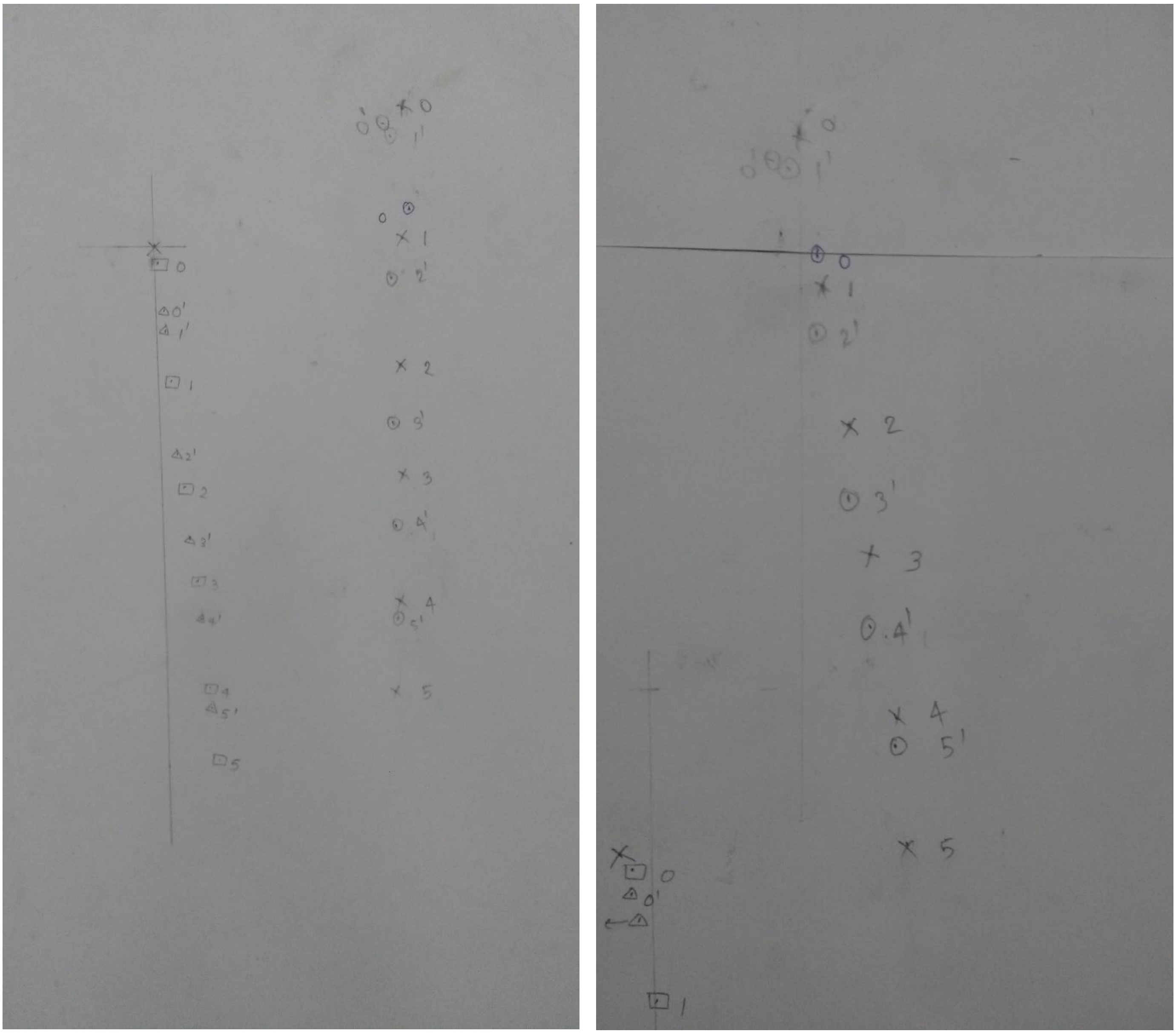
Image 7

Setup and Experimentation

The white board was aligned in a plane parallel to the longitudinal plane passing through the pivot and centre of the chassis. This board was at a distance of 3m front of the central plane. The board was kept in position as shown in the image above with the help of bricks. This board was kept vertical.

After the board and vehicle were placed, the lasers were first positioned at the desired locations. Their height was measured from the ground. After the lasers were put on, the laser was further adjusted to get a starting point on the screen that is at the same height as that of the laser so that any deflection further can be calculated with respect to this position simply be forming a right angle triangle and using geometry.

At this point the setup was done, and we started the experiment by applying a torque that was enough to eliminate the plays in the systems. The preloaded torque was the weight of the extended part of the lever beam and the weight added at a particular distance from the pivot. The position of the laser at this point was taken as the first reading. The deflection of laser at the screen was very minimal and hence we still could consider a right angled triangle for calculation, as also the deflection on screen was negligible with respect to the distance of the screen from the pivot. Further on, weights were added to apply torque in steps and readings were noted for loading condition. Weights were then removed in steps and readings were noted for unloading condition. The deflection measured on the screen are shown in the images below.



a) Points plotted due to deflection at FBH. b) Points plotted due to deflection at front axle location.

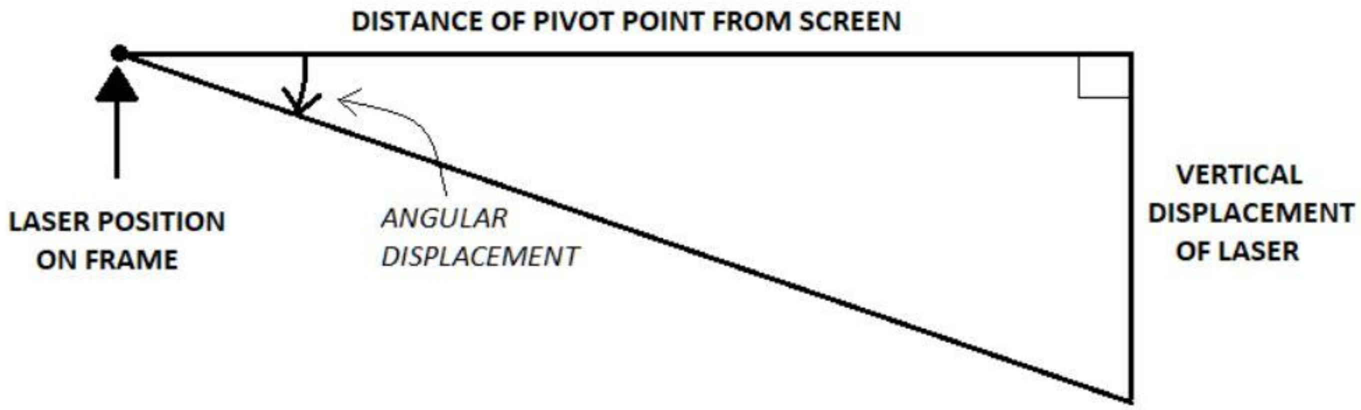
Image 8

In the images above, during the first test the loading was marked by cross 'X' and unloading was marked by circle 'O'. During second test the loading was marked by square '□' and unloading as triangle 'Δ'. Above images are just two reading sets just for example.

At this point the experiment was done and the data collected was used to do further calculations.

Data Collection and Calculation

The measured data included the vertical displacement of the lasers, various geometries of the system, and the force applied to the lever arm. Using these values, the angular deflection was calculated using basic trigonometry. The triangle used to calculate the angular deflection is shown in Figure 13.



As shown in the figure above the angular deflection can be calculated to be:

$$Angular\ deflection = \theta = \tan{-1}\ (vertical\ displacement\ of\ laser/Distance\ from\ laser\ to\ board)$$

After calculating the angular deflection, these points were then plotted against the torque applied to the frame. The torque applied to the frame can be calculated from below equation.

$$Torque = T = moment\ arm * force\ applied$$

In this case, the moment arm is taken to be the distance between the applied load and the pivot point.

Data Presentation

The results of first torsional stiffness test are below. For the first torsional test three lasers were used and their positions were as shown in previous Image 6. We considered the following data which is tabulated below. This table contains the torque and angular deflection for the lasers mounted at the front bulk head (denoted as laser A) and at the front axle (denoted as laser B). The laser B is in more of our interest as it is in the plane where the suspension forces actually acts in and where the torsional stiffness is calculated from. For calculating the torsional stiffness, we used the best curve fit cut method (regression). The mathematical model for the experiment is slightly different from the analytical model in a sense that preload wasn't considered in the above model. The equation used for the experimental data is in consideration with the preloaded torque.

$$T = k * \theta + T_0$$

Here, T_0 = Preload torque i.e. the torque applied for which angular displacement is zero (i.e. θ=0)

k = torsional rigidity.

Torsion test data and results

TORSIONAL STIFFNESS TEST-1

Sr. No.	Mass (kg)	Dist. From pivot (m)	Torque (Nm)	Displ. A (mm)	Theta A (degree)	Displ. B (mm)	Theta B (degree)
1A	6.2	2.228	135.511	0	0	0	0
1B	6	1	194.371	0	0	0	0
2	17.7	2	541.645	36	0.69446	29.6	0.57062
3	18.1	2	896.767	72	1.38872	57.5	1.10912
4	18.4	2	1257.775	106	2.04403	83.5	1.61042
5	30.2	1.5	1702.168	148.5	2.86241	118	2.2752
6	17.6	2	2047.48	179.5	3.45862	144.5	2.78543
7	-17.6	2	1702.168	154	2.96823	123.5	2.38113
8	-18.4	2	1341.16	123	2.3715	99.5	1.91879
9	-18.1	2	986.038	89.5	1.72607	73	1.408
10	-30.2	1.5	541.645	46	0.88734	39.5	0.762
11	-17.7	2	194.371	7	0.13504	6.5	0.1254

By best curve fit method, slope of torque vs theta line was found as follow,

Y – torque

X - theta

$\Sigma y = m\,\Sigma x + c*n$ ... (1)

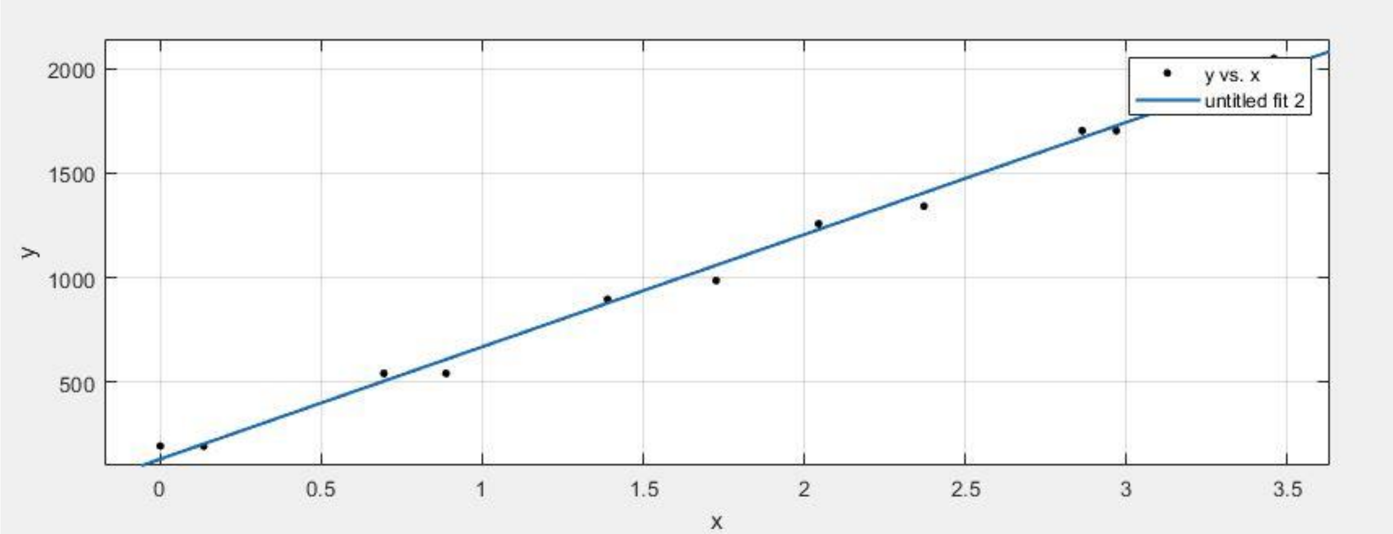
$\Sigma xy = m\,\Sigma x^2 + c\,\Sigma x$ ... (2)

Where, m = slope i.e. torsional stiffness,
c = y-intercept i.e. preloaded torque for zero position,
n=11 i.e. no. of reading,

From above data,

Sr. No.	Quantity	Laser A	Laser B
1	Σx	18.53645	14.861303
2	Σx^2	44.96383	28.60716
3	Σy	11405.588	11405.588
4	Σxy	26588.07674	21204.04349
5	m	536.636	679.410413
6	c	132.6645	118.9695

The figure bellow shows the plot of torque versus angular deflection.



The value of torsional stiffness resulted from first test was 679.4 Nm/degree.

The simulated value of TR was 1124 Nm/degree and measured value was 679.4 Nm/degree. This gave an absolute relative error of 39.55% which is too much more than the allowable 10%. Hence we required to investigate the huge difference/error. On investigating we found that during the experiment, while the front part frame was being twisted there was a slight twist in the rear part of the frame as well which was not considered or calculated since it wasn’t expected to twist as the rear hubs were fixed. Hence, one more test had to be conducted considering the twist in the rear part of the frame as well.

TORSIONAL STIFFNESS TEST-2

For the second test we placed one more laser at the rear bulkhead to measure the angular deflection at the rear, the image for the same is shown in image 6 and the position of the laser is shown in red. This laser is in same plane as that of the rear axle, as this is where the forces act on the rear suspension. The results of the second test were as below. The setup and instrumentation were kept same.

Sr. No.	Mass (kg)	Wt. (N)	Dist. From pivot (m)	Torque (Nm)	Displ. Laser 1 (mm)	Theta 1 (degree)	Displ. Laser 2 (mm)	Theta 2 (degree)	Displ. Laser 3	Theta 3	Displ. Laser4	Theta 4
1A	6.2	60.822	2.228	135.511	0	0	0	0	0	0	0	
1B	5.96	58.468	1	193.9786	0	0	0	0	0	0	0	0
2	17.7	173.64	2	541.2526	34	0.6537	27.5	0.5287	22.5	0.4326	9	0.173
3	18.1	177.56	2	896.3746	68	1.3072	54	1.03813	44.5	0.8552	15.5	0.298
4	18.4	180.51	2	1257.3826	99	1.9028	80	1.5378	66	1.2688	20.5	0.3941
5	30.22	296.46	1.5	1702.0699	137	2.6322	109	2.0948	91	1.7491	25.5	0.4903
6	17.99	174.52	2	2051.1097	164	3.15	129.5	2.4883	108	2.0756	28.5	0.5479
7	-17.79	296.46	2	1702.0699	144	2.766	114	2.1908	89	1.7107	27.5	0.5287
8	-30.22	180.51	1.5	1257.3826	112.5	2.162	87.5	1.6819	66	1.2688	24.5	0.471
9	-18.4	177.56	2	896.3746	85	1.6338	66	1.2688	45	0.8651	22.5	0.4326
10	-18.1	173.64	2	541.2526	57	1.0958	42	0.8075	23	0.4422	19.5	0.3749
11	-17.7	60.822	2	193.9786	20	0.3845	10	0.1923	-6	-0.1154	9	0.173

TORSIONAL STIFFNESS VALIDATION RIG

Experimental Results: -

Preloaded mass=6.2kg
Distance from pivot=2.228m.
Preloaded torque =6.2X9.81X2.228= 135.511416 N.m

Preload Torque = 193.9786 Nm

Distance of screen from Centre/pivot = 2.98 m

LOADING CONDITION

Sr no.	Mass (kg)	Wt. (N)	Dist. Frm pivot (m)	Torque (N-m) 193.9786 (135.5114)	(1) Laser A Def. (mm)	θ_A (deg.)	(2) Laser B Def. (mm)	θ_B (deg.)	(3) Laser C Def. (mm)	θ_C (deg.)	(4) LD Def. (mm)	θ
1	5.96	58.4676	1m	+58.4676	0	0	0	0	0	0	0	0
				+347.274								
2.	17.7	173.637	2	541.2526	34	0.6537	27.5	0.5287	22.5	0.4326	9	0.173
				+355.122								
3	18.1	177.561	2	896.3746	68	1.3072	54	1.03813	44.5	0.8552	15.5	0.298
				+361.008								
4	18.4	180.504	2	1257.3826	99	1.9028	80	1.5378	66	1.2688	20.5	0.3941
				+444.6873								
5.	30.22	296.458	1.5	1702.0699	137	2.6322	109	2.0948	91	1.7491	25.5	0.4903
				+349.0398								
6	17.79	174.5199	2	2051.1097	164	3.15	129.5	2.4883	108	2.0756	28.5	0.5479

UNLOADING CONDITION

Sr no.	Mass (kg)	Wt. (N)	Dist. Frm pivot (m)	Torque (N-m)	Laser A Def. (mm)	θ_A (deg.)	Laser B Def. (mm)	θ_B (deg.)	Laser C Def. (mm)	θ_C (deg.)		
				−349.0398								
7	−17.79		2	1702.0699	144	2.766	114	2.1908	89	1.7107	27.5	0.5287
				−444.6873								
8	−30.22		1.5	1257.3826	112.5	2.162	87.5	1.6818	66	1.2688	24.5	0.4710
				−361.008								
9	−18.4		2	896.3746	85	1.6388	66	1.2688	45	0.8651	22.5	0.4326
				−355.122								
10	−18.1		2	541.2526	57	1.0958	42	0.8075	23	0.4422	19.5	0.3749
				−347.274								
11	−17.7		2	193.9786	20	0.3845	10	0.1923	−6	−0.1154	9	0.173
				−58.4676								
12	−5.96		1	135.5114	14	0.2692	5	0.09613	−12	−0.2307	7	0.1346

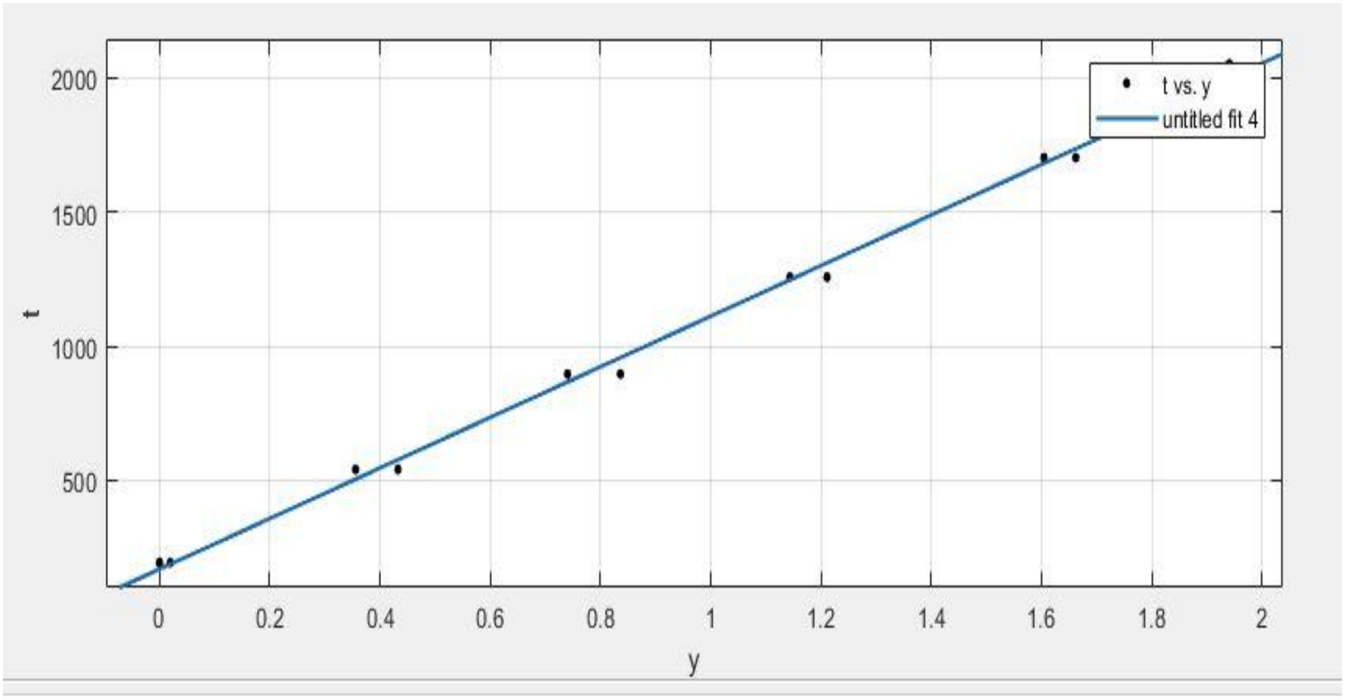
The table below gives the values of torques and corresponding angular deflection for all four lasers at different positions in the first five columns. The last three columns give the relative angular displacement with respect to the laser at the rear i.e. laser D (or 4). For example, θ1'= θ1- θ4, so on and so forth.

Torque	θ 1	θ 2	θ 3	θ 4	θ 1’	θ 2’	θ 3’
193.9786	0	0	0	0	0	0	0
541.2526	0.6537	0.5287	0.4326	0.173	0.4807	0.3557	0.2596
896.3746	1.3072	1.03813	0.8552	0.298	1.0092	0.74013	0.5572
1257.3826	1.9028	1.5378	1.2688	0.3941	1.5087	1.1437	0.8747
1702.0699	2.6322	2.0948	1.7491	0.4903	2.1419	1.6045	1.2588
2051.1097	3.15	2.4883	2.0756	0.5479	2.6021	1.9404	1.5277
1702.0699	2.766	2.1908	1.7107	0.5287	2.2373	1.6621	1.182
1257.3826	2.162	1.6819	1.2688	0.471	1.691	1.2109	0.7978
896.3746	1.6338	1.2688	0.8651	0.4326	1.2012	0.8362	0.4325
541.2526	1.0958	0.8075	0.4422	0.3749	0.2115	0.4326	0.0673
193.9786	0.3845	0.1923	-0.1154	0.173	0.1346	0.0193	0.2884

Results

Linear model Poly1:
f(x) = p1*x + p2
Coefficients (with 95% confidence bounds):
p1 = 942 (898, 986.1)
p2 = 169.5 (120.8, 218.2)

Goodness of fit:
SSE: 1.517e+04
R-square: 0.9962
Adjusted R-square: 0.9957
RMSE: 41.06



The value of torsional stiffness resulted from second test was 942 Nm/degree.

The simulated value of TR was 1129 Nm/degree and measured value was 942 Nm/degree. This gave an absolute relative error of 16.2%. The acceptable value should lie within 10% relative percentage error while 20% error is avoidable. So, a value with 16.2% lies between these values, so this value is also acceptable for now with further investigation to be done.

Conclusion

As the chassis gets less stiff the vehicle behaves in an increasing undesirable manner to driver inputs, and becomes increasing hard to drive. vehicles should be designed for the lightest chassis possible that has a torsional stiffness greater than a target value, 1400 Nm/deg in our case, with further research required on the targeted value. Stiffnesses greater than this value result in added weight for diminishing returns, while stiffnesses less than this value put the car in danger of entering a regime in which the stiffness has significant effect on vehicle behavior.